On recognizing an object with a partition

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Abstract

Smith and Brogaard ('A unified theory of truth and reference' *Logique et Analyse* 43 (2003) 49-93) proposed a resolution of the problem of referential ambiguity based on the use of mereotopological partitions. It is shown that this proposed resolution is circular if viewed ontologically and intractable if viewed epistemologically.

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1 Introduction

Quine (1960, 1969) can be interpreted as holding that nothing connects a word to an object beyond the speaker's assumption - shared one hopes by at least some listeners - that the word can be understood to refer to the object. Smith and Brogaard (2003) propose to do better than this, even in the presence of referential vagueness, by appealing to the notion of a *partition* that "recognizes" not just a particular singular object John but "*all* of the aggregates f_i that are almost identical to John" (p. 79; emphasis in original). This sense of recognition is formalized in terms of the "location" of an object x within a partition A:

$$x \in A =_{def} \exists z(L_A(x, z)), \tag{1}$$

where ($L_A(x, z)$) indicates that "x is located in cell z of A." Smith and Brogaard consider objects to be *bona fide* real, objective entities[1] that exist "independently of any acts of human fiat and independently of our efforts to understand (them) theoretically" while partitions and the cells that they comprise are "artefact(s) of our judging, classifying, theorizing, or mapping activity" (p. 74); they claim, however, that "once a given partition exists, it is ... an objective matter whether or not that object is located in that cell" (p. 76). Recognition of an object by a partition, therefore, resolves the "mystery of reference" *objectively* for Smith and Brogaard.

I show here that any objective reading of the notion of recognizing an object with a partition is either circular or intractable. Briefly, the notion is circular if viewed ontologically, as the definition (1) of recognition implicitly assumes that the stipulated "fiat" partition A corresponds exactly to the objective mereotopology of the "aggregates" f_i that are to be recognized; A therefore accomplishes nothing not already done by the assumed objective mereotopology. The notion is intractable if viewed epistemologically, as the finite but arbitrary number of the f_i render it impossible to determine using finite means whether (1) is satisfied for a given object and a given partition. The idea of a "consistent history" of partitions that Smith and Brogaard (2002) develop to account for the possibility of consistently referring to an object that endures through time depends on the notion of recognition and is, therefore, also either circular or intractable. As Smith and Brogaard adapt this idea of a consistent history from its application to the interpretation of quantum mechanics[2], I comment briefly on an explicitly quantum-mechanical formulation of recognition with a partition, and show that this formulation, like that of Smith and Brogaard, in fact provides no advance over the Quinean position that shared terms must simply be assumed to refer to their target objects.

2 Partitions as truthmakers

Smith and Brogaard introduce partitions into natural-language semantics in order to cope with the well-known problem of referential vagueness: a term such as "Mont Blanc" can refer to many different circumscriptions of the world, some of which contain the rabbits living on the slopes of the famous mountain, while others do not. Uses of the term "Mont Blanc" all, however, occur in particular contexts that render relevant different aspects of these various circumscriptions; the resident rabbits are irrelevant when Mont Blanc is pointed out from

[1]Smith and Brogaard (2003) give Quine his due by allowing that reality may be "intrinsically undifferentiated as far as metaphysical distinctions and categories are concerned" (p. 87). However, they use terms like "Mont Blanc" and "rabbits" throughout with the assumption that such things have real boundaries that objectively differentiate them from the other furniture of the universe. Smith and Brogaard (2002) and Grenon and Smith (2008) include explicit statements of the common-sense realism about ordinary objects that is implicit throughout Smith and Brogaard (2003).

[2]Smith and Brogaard (2003) cite the "Consistent Histories" interpretation of Omnès (1994) as an inspiration for their notion of an object's "location" within a cell; Smith and Brogaard (2002) provide a more explicit review of the origin of their idea of a "history" in the quantum mechanics literature. See Griffiths (2011) for a recent exposition of this approach within quantum theory.

afar, but are relevant if its ecology is being discussed. Whether the rabbits are relevant depends on the *granularity* of the context: its demand for details, and its concommittant provision of the opportunities and technologies needed to observe the demanded details[3]. Partitions control granularity by enforcing a finite limit on the size or scope of each cell; associating a partition with a context of discourse blocks mereotopological regresses and hence blocks inferences such as:

John sees Mont Blanc.

Mont Blanc includes numerous rabbits.

∴ John sees numerous rabbits.

Hence while *reality*, even from the perspective defined by a given context, does not make all intuitively true sentences true and false sentences false, reality plus a contextually-appropriate partition does. By limiting the scope of discourse in any given context, partitions become truthmaking overlays on reality.

Thrusting partitions into the role of truthmakers clearly raises two questions. The first is ontological: even if we acknowledge that they are fiat entities created for the purpose by us, do the "right" partitions to do the job of truthmaking exist? The second question is epistemological: is it possible, in a given context, to know that a given partition will do the needed truthmaking job? Smith and Brogaard deal explicitly with neither of these questions, apparently assuming that an appropriate partition can be found or simply stipulated for any context. If it is "an objective matter" whether a proferred partition provides a truthmaker, however, this assumption is problematic, because it is an assumption that *fiat* can always be matched with *physics*, that is, with the actual mereology of the real world.

Addressing the ontological question requires returning to the problem that partitions are meant to resolve, the problem of ambiguous reference. In the actual mereology of the real world, the objects of ordinary experience are complex entities comprising vast numbers of parts: hunks of ice and rock in the case of Mont Blanc, living cells in the case of the rabbits, molecules, atoms and elementary particles in the case of all material objects[4]. Under ordinary circumstances, reference is ambiguous because a precise accounting of such parts is neither demanded nor made; contexts shifts are significant because they can introduce both requirements to account for such parts and technologies that enable doing so. To employ an example of Smith and Brogaard's, whether a water glass is empty has a different

[3]Smith and Brogaard (2003) introduce granularity as a spatial concept; however, they employ it as a general term for distinguishing larger from smaller scopings of relevant facts independently of the dimensions along which "scope" is defined.

[4]Ontologies can be constructed in which such part-hood relations are discounted; that of Cartwright (1999) is a case in point. Smith and Brogaard (2003) do not adopt such an ontology, and claim explicitly that "Elite things and classes are in our terms the things and classes captured by those partitions which track bona fide boundaries and relations in reality. It is the job of science to move us in the direction of partitions of this sort" (p. 90).

answer for a pathologist with a microscope than it has for someone who has just drained it of water. An enduring object of reference x, then, is not a simple, but rather a finite collection of "aggregates" $f_1, f_2, ..., f_n$, where for any scale of parthood significantly smaller than that of x itself n is enormously large[5]. If quantum-mechanical effects are ignored, then at any given instant x can be taken to be identical to some particular f_k; however, for an enduring x the particular aggregate to which x can be regarded as identical changes from moment to moment as ice forms and evaporates, cells are born and die, atoms drift off and are re-captured. Hence if a partition A is to recognize x as an enduring object of reference, it must recognize within the same cell z *all* of the f_i as Smith and Brogaard require; the collection $\{f_i\}$ must, as Grenon and Smith (2008) require explicitly, be an equivalence class for $L_A(x, z)$.

Capturing all of the "almost identical" aggregates f_i is, however, insufficient to assure the fidelity of reference. If A is to serve as a referentially-transparent truthmaker for judgments regarding x, its cells must be large enough to include the f_i, but not so large as to also contain recognized non-x's. Hence the minimal cell z that contains x, i.e. all of the f_i, must not contain anything else recognized by A, i.e. it must be the case that:

$$x \in A \Longrightarrow \exists z (L_A(x, z) \land \forall y \neq x, (y \notin A \lor \neg L_A(y, z))), \tag{2}$$

where as in Smith and Brogaard, '$p \Longrightarrow q$' abbreviates 'it is not possible for p to be true and q to be false.' Hence if the resident rabbits are not, as Smith and Brogaard claim, part of Mont Blanc, any partition that recognizes the rabbits must put them in a different cell from Mont Blanc.

Consider an aggregate g of parts at some scale, such that $g \notin \{f_i\}$; such an aggregate must exist if x is smaller than the entire universe. Assuming that $x \in A$ and that z is the minimal cell for which $L_A(x, z)$, (2) requires that:

$$L_A(g, z) \Longrightarrow g \notin A, \tag{3}$$

i.e. that g is "invisible" with A. One can now ask, what partitions A *guarantee* that (3) holds for all g? One partition clearly satisfies this requirement: the "natural" partition N that corresponds exactly to the actual mereology of the real world. Suppose now that $A \neq N$ satisfies (3) for all g, and suppose that $g = f_k \oplus \zeta$, where f_k is one of the aggregates recognized by A as Mont Blanc and ζ is a nearby cubic centimeter of air. The partition A must, in this case, be such that Mont Blanc - at least in its guise as f_k - disappears from referential view if it is aggregated with a bit of the local air. This clearly makes no sense: the point of introducing partitions is to resolve such problems with referential ambiguity. Expressions (2) and (3), which accord with common sense when objects y that are ordinarily regarded as fully distinct from x are considered, grossly violate common sense when "unnatural" aggregate objects y that overlap with x are considered. In order to avoid such cases, it seems that (2) must be replaced by a more restrictive condition:

[5]Clearly n cannot be infinite, or the cell would contain the entire universe.

$$x \in A \Longrightarrow \exists z(L_A(x,z) \wedge \forall y \text{ such that } y \cap x = \emptyset, (y \notin A \vee \neg L_A(y,z))), \qquad (4)$$

where '$y \cap x = \emptyset$' here indicates that x and y are placed in different cells by the natural partition N.

The difficulty, clearly, is that (4) amounts to a circularity: A does no work in (4) that is not already accomplished by N. The natural partition is, however, *not* established by fiat; it is established by the laws of nature. Once appeal is made to (4) to specify what each cell of a truthmaking partition must recognize, the entire apparatus of fiat partitions as truthmakers is rendered redundant. If N is available, reality itself is the truthmaker.

3 Identifying a truthmaking partition

Let us set aside the question of whether N exists - Smith and Brogaard clearly assume that it does - and suppose that some partition A has been put forward as a truthmaker for judgments about x in some context. Is it possible to determine whether A is in fact a truthmaker for such judgments in that context, i.e. is it possible to determine the "objective matter (of) whether or not that object is located in that cell" of A? From the reasoning above, this is clearly the question of whether it is possible to determine whether some cell z of A contains all of the f_i corresponding to the object of reference x and contains no non-x's that are recognized by A.

If one thinks of A as a "theory" of x, then the question above becomes the question of empirical theory confirmation, a question with a well-known negative answer. This can be made precise as follows. Consider a finite sequence of observations of x made at times t_i, for simplicity keeping the context and means of observation fixed. Neglecting quantum-mechanical effects as above, x can be considered to be identical to some f_k at t_k, in which case x can be considered to be a classical finite state machine[6] that executes state transitions $...f_i \rightarrow f_j \rightarrow f_k...$ in the intervals between the observation times $...t_i, t_j, t_k...$ Theorem 2 of Moore (1956) shows that no finite sequence of observations of such a machine suffices to establish the identity of the machine, i.e. its complete set of states and the transition rule that governs its behavior. It is therefore the case that no finite sequence of observations of x is sufficient to determine whether any specified cell z of a partition A captures all of its "almost identical" aggregates f_i; clearly it is similarly impossible to establish with a finite sequence of observations that z captures nothing but the f_i. Partitions are in this sense like theories: their sufficiency as truthmakers cannot be determined empirically.

[6]For definitions and examples see Ashby (1956) or Hopcroft and Ullman (1979).

4 Consistent histories

Intuitively, the sequence of particular aggregates f_i to which an object of reference x can be regarded as identical at a sequence of observation times t_i corresponds to a "history" of x at the granularity of the t_i. Smith and Brogaard (2002) formalize this intuition using changes in spatial location as a metaphor for changes in observation context or in the values returned by some ancillary measurement carried out on x at each of the t_i. They consider a partition A with cells corresponding to locations (in their motivating example, airports), observation contexts or measurement outcomes, and define a *history* of x as a sequence of cells of A indexed by time, and hence a sequence of propositions $L_i(x, z_i)$ where the index i ranges over observation times (Smith and Brogaard (2002); p. 4). As x is, as a matter of objective fact described by N, identical to some aggregate f_i at every t_i, this definition adds to the intuition above only the extra "dimensions" of location, context, or measurement outcome. A history is *consistent* if its component sentences $L_i(x, z_i)$ are all mutually consistent. Consistency obtains on this model, clearly, whenever x avoids occupying two distinct cells at the same time.

Smith and Brogaard introduce consistent histories in order to provide an "extension of the mereotopological ontology to deal with change and becoming" (Smith and Brogaard (2002) p. 8) that treats macroscopic objects as enduring aggregate entities and maintains the intuitive, qualitative distinction between space and time. While their account of partitions as truthmakers does not explicitly rely on this notion of historical consistency, by treating objects such as Mont Blanc or its rabbits and actions such as John kissing Mary in a straightforward, ordinary-language way it does so implicitly. The notion of a consistent history depends, conversely, on the ability of partitions to function as truthmakers, i.e. on the ability of '$L_i(x, z_i)$' to fully and unambiguously capture x within z_i at t_i. By basing both on the ability to recognize an object with a partition, Smith and Brogaard render a straightforward ontology of enduring bounded objects and a straightforward semantics of true sentences referring to such objects co-dependent.

The ontological and epistemological arguments rehearsed above apply, *mutatis mutandis*, to the concept of a consistent history. The ontological argument shows that this concept is circular: the only history that assures consistency is a history generated by extending N to encompass not only all objects but all locations, contexts and measurements within an assumed actual mereology of the real world. Only this history assures that x, in its many manifestations as particular aggregates, never occupies two locations simultaneously, and only this history assures that distinct objects x and y never occupy the same location. If this N-based history is assumed, however, the construction of further histories is pointless; any fiat history based on a fiat partition A does only some of the work of the N-based history, and does it only as a mereological approximation. Such approximations are clearly valuable as practical science, but they cut no ice as *ontology.* The epistemological argument simply reinforces this point, by showing that the consistency of a proferred fiat history based on a fiat partition A can never be demonstrated.

5 A note on quantum-mechanical partitions

The foregoing has explicitly neglected quantum-mechanical effects; the assumption that an observed system x can be regarded as identical at t_i to a *particular* aggregate f_i of elementary parts is, in particular, inconsistent with the superposition principle of quantum mechanics, which requires that any linear combination of states of x is itself a state x[7]. Smith and Brogaard (2002) borrow the formal notion of a consistent history from quantum mechanics; they title their paper 'Quantum mereotopology'. The point of consistent histories within quantum mechanics is to provide an "interpretation" of the formalism that solves the problem of the "emergence" of a classical world of well-defined objects that can be observed in well-defined states[8]. Might a purely quantum-mechanical process provide the underlying ontological basis for N, and hence for a theoretical solution to the problems outlined above?

In quantum mechanics, a consistent history is a sequence of mutually-commuting measurements to which a well-defined probability can be assigned[9]. Measurements are operators defined on the Hilbert space of a quantum system; traditionally measurements were taken to correspond to orthonormal sets of projection operators, one associated with each distinct possible outcome, while more recently the orthogonality requirement has been dropped and any positive operator-valued measure (POVM), i.e. any normalized set of positive semi-definite operators on the Hilbert space, where again each operator is associated with an outcome, is regarded as a measurement[10]. A consistent history of a system x as it evolves through time can, then, be thought of as a sequence of outcomes, each associated with some time t_i, of operations with components of some POVM on the Hilbert space of x. Defining such a sequence clearly requires specifying the Hilbert space of x, i.e. it requires specifying a partition that distinguishes the degrees of freedom of x from the degrees of freedom of the rest of the universe. A consistent history in quantum mechanics thus requires a quantum-mechanical partition of (universal) Hilbert space, just as a classical consistent history requires N.

Quantum mechanical systems can be isolated, and hence partitioned from the rest of the universe, "for all practical purposes" in the laboratory. *Ontologically* isolating a quantum system, however, involves violating the superposition principle as applied to the quantum state of the universe as a whole. Griffiths (2011) avoids this problem by building Hilbert-space partitions into the mutually-incompatible "frameworks" within which consistent quantum-mechanical statements can be formulated. As no framework allows all true

[7]The superposition principle motivates the choice of vectors in Hilbert space as the mathematical representation of physical states and is typically considered as axiomatic in quantum mechanics. See http : //en.wikipedia.org/wiki/Quantum_mechanics or for a briefer, more axiomatic introduction http : //plato.stanford.edu/entries/qm/.

[8]The traditional "measurement problem" is a special case of the general problem of the emergence of classicality in a fundamentally quantum world. Schlosshauer (2007) provides a textbook-length introduction; Landsman (2007) or Wallace (2008) are good general reviews.

[9]See Griffiths (2002) Ch. 10, Griffiths (2011) or relevant references in Smith and Brogaard (2002).

[10]For an introduction to POVMs, see Nielsen and Chaung (2000) Ch. 2. That the POVM formalism can be consistently applied in a mereologically-nihilist context is shown in Fields (2012).

statements to be formulated, this solution effectively rejects "the quantum state of the universe" as meaningless. Not having a meaningful quantum state, however, means not having a meaningful Hilbert space, i.e. not having a meaningful collection of degrees of freedom. Griffith's solution, therefore, rejects the idea that there is a actual mereology of the real world; hence is rejects the self-consistency of N.

6 Conclusion

By providing an adjustable level of granularity with a natural mereotopological interpretation, the formal notion of a partition offers an attractive approach to the problem of referential ambiguity. What has been shown here is that this approach cannot be pushed to the limit of full objectivity. Any fiat partition that is overlaid onto the world is itself referentially ambiguous in a way that cannot be repaired without appeal to an hypothesized "natural" mereology N, and cannot be demonstrated to be repaired even if N is assumed. Hence fiat partitions cannot serve as *objective* truthmakers, however useful they may be as "for all practical purposes" truthmakers.

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