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Motion as manipulation: Implementation of force-motion analogies by event-file binding and action planning

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Abstract:

Tool improvisation analogies are structure-mapping inferences implemented, in many species, by event-file binding and pre-motor action planning. These processes act on multi-modal representations of currently-perceived situations and eventuate in motor acts that can be directly evaluated for success or failure; they employ implicit representations of force-motion relations encoded by the pre-motor system, and do not depend on explicit, language-like representations of relational concepts. A detailed reconstruction of the analogical reasoning steps involved in Rutherford's and Bohr's development of the first quantized-orbit model of atomic structure is used to show that human force-motion analogies can in general be implemented by these mechanisms. This event-file manipulation (EFM) model of the implementation of force-motion analogies is distinguished from the standard view that structure mapping analogies require the manipulation of explicit, language-like representations of relational concepts.

Keywords: Structure mapping; Tool improvisation; Rutherford-atom analogy; Pre-motor system; Mirror-neuron system; Physical reasoning; Conceptual reasoning

Introduction

The ability to recognize similarities at the level of relational structure between remembered and novel situations, and hence to reason by structure-mapping analogy, is foundational to human intelligence (reviewed by Markman and Gentner, 2001; Gentner, 2003; Holyoak, 2005) and to the practice of science in particular (reviewed by Holyoak and Thagard, 1995; Feist and Gorman, 1998). In the almost three decades since the introduction of structure mapping as a formal model of analogy by Gentner (1983), a “standard view” of structure-mapping inference as mechanistically dependent on the recognition of explicit lexical symbols representing relational concepts, and hence as a human-specific inferential capability, has come to dominate the analogy research community. This standard view has, however, recently been challenged by the observation that tool-improvisation analogies, which are carried out in the wild by many non-human animals, involve structure-mapping inferences over force-motion relations (Fields, 2011a).

The present paper further investigates the human implementation of structure-mapping inferences involving forces and motions using the well-known “Rutherford atom analogy” electrons:nucleus::planets:sun (e.g. Falkenhainer *et al.*, 1989; Forbus *et al.*, 1994; Gentner and Wolff, 2000; Green *et al.*, 2006; Dietrich, 2010) as an example. On the standard view, the Rutherford atom analogy is organized around the explicit relational predicates REVOLVE-AROUND(x,y) and ATTRACTS(x,y), which drive a structure mapping inference from the base case REVOLVE-AROUND(planets,sun) and ATTRACTS(sun,planets) to the target case REVOLVE-AROUND(electrons,nucleus) and ATTRACTS(nucleus,electrons) (e.g. Falkenhainer *et al.*, 1989, Fig. 9; Forbus *et al.*, 1994, Fig. 6; Green *et al.*, 2006, Fig. 1). Such reasoning is, however, not to be found in Rutherford's 1911 paper presenting his novel model of the atom; what one finds instead are a variety of other analogies, the most prominent and theoretically productive of which is alpha-particle:central-charge::electron:atom (Rutherford did not use the term “nucleus” in 1911). Both this analogy and the others that appear explicitly in Rutherford (1911) are surprisingly similar to tool-improvisation analogies. It is shown here that these latter analogies can be implemented using the representational and inferential apparatus provided by the “event-file manipulation” (EFM) model developed in Fields (2011a). Based on these EFM model implementations, it is suggested that electrons:nucleus::planets:sun is derivative from alpha-particle:central-charge::electron:atom, and that it is compelling not because it rests on the manipulation of concepts such as REVOLVE-AROUND, but because it rests on an ancient and ubiquitous motor memory of orbital motion. The present analysis thus supports the general view of cognition as essentially embodied (Lakoff and Johnson, 1999), the hypothesis that concepts derive at least in part from perceptual-motor simulation (Barsalou, 1999; 2008; Galese and Lakoff, 2005), and the recent re-conceptualization of the pre-motor system as a general-purpose event prediction system (Butz, Sigaud and Gérard, 2003; Schubotz, 2007; Bubic, von Cramon and Schubotz, 2010).

The paper is organized as follows. The next section, “Background: The 'standard view' of structure mapping and its alternatives” outlines the standard view of analogy and its assumptions regarding the implementation of structure mapping, and contrasts these assumptions with those of published alternatives including the EFM model. The third section, “Rutherford's 1911 reasoning and the 'Rutherford atom analogy'” reviews the force-motion inferences that appear in Rutherford (1911) and Bohr (1913), and examines how these inferences relate to electrons:nucleus::planets:sun. The fourth section, “Force-motion analogies in the EFM model” reviews the assumptions of the EFM model, and then employs it to reproduce Rutherford's published reasoning. The paper concludes by suggesting that structure mapping is an ancient ability, and that its appearance in animals faced by problems involving forces and motions should not be considered surprising.

Background: The “standard view” of structure mapping and its alternatives

Structure mapping inferences are widely regarded as mechanistically dependent on one particular way of recognizing similarities in relational structure between situations: the recognition of explicit lexical symbols representing relational concepts that are common to the encodings of disparate situations (e.g. Fodor, 2000; Gentner, 2003; Holyoak, 2005; Penn, Holyoak and Povenelli, 2008). For example, the Structure Mapping Engine (SME), the first full-scale implementation of structure mapping, represented relational concepts as explicit, multi-argument predicate symbols such as CAUSE(x,y) or GREATER-THAN(x,y), with the proviso that “relations must always match identically” for inference to occur (Falkenhainer, Forbus and Gentner, 1989, p. 10). The later Many Are Called/Few Are Chosen (MAC/FAC) system (Forbus, Gentner and Law, 1994) similarly employed exact matches between relational predicates, both in the “MAC” stage of counting occurrences of lexical symbols and the “FAC” stage of computing structure mappings using SME. The hybrid symbolic-connectionist Learning and Inference with Schemas and Analogies (LISA) system (Hummel and Holyoak, 2003) maintained the lexical representation of relational predicates, but replaced exact symbol matching with constraint propagation through an experience-dependent network to determine the similarity between relations appearing in representations of different situations. Dietrich (2010) similarly maintained the lexical representation of relational predicates, but replaced exact symbol matching with an obligate abstraction and re-categorization process. The idea that structure mapping is mechanistically dependent on the recognition of explicit lexical symbols representing relational concepts is sufficiently dominant among analogy researchers to be considered the “standard view” of structure-mapping analogy.

Its emphasis on the explicit representation of relational concepts as predicate symbols ties the standard view of structure mapping closely to the idea that the data structures employed by human cognitive processing are implemented in a "language of thought" or "mentalese" that generalizes the structural and semantic capabilities of human natural languages. Gentner (2003) expresses this theoretical link explicitly, claiming that "relational concepts are critical to higher-order cognition … analogy is the key to conceptual learning, and relational language is the key to analogy" (p. 196); similarly, Gentner and Christie (2008) claim that "possession of an elaborated symbol system – such as human language – is necessary to make our relational capacity operational" (p. 136). The human specificity of natural languages is in turn taken to explain, at least in part, the apparent human-specificity of structure mapping. Gentner (2003), for example, claims that non-human animals can perform structure-mapping analogies "only if they learn relational language" (p. 219), while Penn *et al.* (2008) emphasize that "only humans appear capable of reinterpreting the higher-order relation between (these) perceptual relations in a structurally systematic and inferentially productive fashion … only humans form general categories based on structural rather than perceptual criteria, (and) find analogies between perceptually disparate relations" (p. 110). This view of structure-mapping analogy as a distinct form of human-specific, domain-general, language-dependent conceptual reasoning is consistent with, and typically an explicit component of, a traditionally rationalist orientation toward cognition that explicitly separates "central" or "high-level" processes that are conceptualized as formal operations on essentially propositional data structures from "peripheral" or "low-level" processes that are conceptualized as automated operations of architecturally-modular, cognitively-impenetrable, evolutionarily-old components of the brain (e.g. Fodor, 1983; 2000; Pylyshyn, 1986).

As alternatives to the standard view of analogy as human-specific, domain-general and language-dependent, it has been proposed that analogical inferences result from goal-dependent priming of low-level relation recognition (Bar, 2008; Leech *et al.*, 2008), or that both relational concepts and analogical or metaphorical inference are implemented by perceptual-motor simulation (Barsalou, 1999; Lakoff and Johnson, 1999; Gallese and Lakoff, 2005; Barsalou, 2008). It has not been clear, however, 1) whether or how either goal-driven priming or perceptual-motor simulation could implement structure mapping as an algorithm, 2) whether or how these processes could support relation-driven structure mapping in preference to inferences based solely on surface similarity and hence produce analogical inferences meeting criteria of systematicity (Gentner, 2005; Holyoak, 2005), or 3) whether or how such processes could operate in organisms lacking languages that included relational concepts. Thus it has not been clear whether or how neurocognitive processes that did not employ explicit lexical representations of relational concepts, and hence were not explicitly dependent on language – whether overt public languages or "mentalese" - could implement the kinds of language-based analogies typically studied in analogy research (Gentner, 2003; 2005; Holyoak, 2005) and included into the "verbal" sections of intelligence tests. While some computational experiments in cognitive robotics are targeted toward answering this question (e.g. Cangelosi et al., 2010), available data are insufficient to develop a computational architecture that realistically models the primate perceptual-motor system at the level of functional detail required to adequately test the claim that structure-mapping inference can be implemented by perceptual priming or perceptual-motor simulation.

Experimental investigation of the neurocognitive implementation of structure mapping in humans has thus far yielded primarily low-resolution localization data (reviewed by Bar, 2008) that neither confirm nor reject the standard hypothesis that structure mapping and hence analogical reasoning depend on explicitly-represented relational concepts. The recognition of relational structures is known to involve the frontal-parietal working memory (WM) network (Waltz *et al*., 2000; Green *et al.*, 2006) that is generally involved in making long-distance semantic connections (Jung-Beeman *et al*., 2004; Kounios *et al*., 2007; Bar, 2008; Sandkuhler and Bhattacharya, 2008). The mapping step involves regions of polar or rostral prefrontal cortex (Bunge *et al.*, 2005; Morrison *et al.*, 2005; Green *et al*., 2006; Wendelken *et al*., 2008; Knowlton and Holyoak, 2009; Cho *et al*., 2010) that are also implicated in multi-tasking (Dreher *et al*., 2008) and allocating attention between externally-driven perception and internal imaginative processes (Gilbert *et al.*, 2005; Burgess *et al.*, 2007). As do other forms of externally-directed problem solving, efficient analogy formation involves default-network deactivation (Buckner *et al*., 2008; Kounios and Beeman, 2009). However, the number and variety of analogical reasoning tasks that have been investigated is small, the experimental tasks employed induce enhanced activity relative to control tasks in areas other than the regions of interest on which detailed analysis has been focused, and no anatomical structures or functional pathways fully specific to analogy have yet been characterized.

Comparative studies of inferential capabilities across organisms and task environments provide an alternative to computational simulation or neuroimaging methods for investigating the implementation of structure mapping. For example, the inferences supporting tool improvisation require for practical validity the mapping of relations between applied forces and achieved motions from remembered situations to novel situations, typically independently of the surface features of the objects involved; such inferences therefore meet the definitional criteria for structure mapping analogies (Fields, 2011a). Mammals and birds of many species improvise tools; tool improvisation analogies must, therefore, be implemented by neurocognitive mechanisms that do not depend on human-like natural language and, if the preponderance of evidence is to be accepted, do not depend on explicit "mentalese" encodings or conscious awareness of relational concepts (Penn and Povinelli, 2007; Suddendorf and Corballis, 2007; Penn *et al.*, 2008). On the basis of primarily neurofunctional evidence, Fields (2011a) proposed that tool-improvisation analogies are implemented across phyla by event-file binding and action planning systems that are structurally homologous across mammals and appear to be shared as functional systems by mammals and birds. The EFM model formalizes this proposal, showing how the complex relation "Tool A moved along trajectory B will transfer force C to target D, thus achieving goal E" can be implemented by a pre-motor action-planning network capable of retrieving, comparing, and recombining components of memory-resident representations of multiple instances of successful and unsuccessful actions.

The present paper extends the investigation of the implementation of force-motion analogies initiated in Fields (2011a) from tool-improvisation analogies to the general case. It advances two hypotheses: 1) that human beings implement structure mapping inferences involving applied forces and the resulting motions using the mechanisms proposed by the EFM model even in abstract domains; and 2) that the human ability to formulate force-motion analogies using explicit relational concepts is derivative from, not a precondition for, the ability to perform force-motion analogies without the use of explicit relational concepts.

Rutherford's 1911 reasoning and the "Rutherford atom analogy"

Ernest Rutherford's 1911 model of the atom as consisting of a small central nucleus surrounded by electrons was proposed to account for the results of experiments in which gold atoms were bombarded by high-energy alpha particles, products of radioactive decay that were known at the time to have the mass of helium atoms. Most of the alpha particles went straight through the thin gold foil target as expected on the basis of earlier experiments, but one in 20,000 were deflected at large angles, suggesting collisions with a small, dense central object and thoroughly contradicting the then-dominant Thompson or "plum pudding" model of atoms as spheres containing electrons embedded within a uniform positively-charged material (Rutherford, 1911; Randall, 2005 briefly reviews the relevant history from a physicist's perspective; Mehra and Rechenberg, 1982 provide a more detailed historical review). Rutherford's model was revolutionary in that it proposed an atom consisting mostly of empty space, in which the positive charges were concentrated in the center and the negative charges (the electrons) occupied the distant periphery.

What is now called the "Rutherford atom analogy" was not the first analogy that Rutherford employed in trying to understand the astonishing experimental result of alpha particles being deflected backwards by a gold foil. To colleagues, he described the result as analogous to an artillery shell being fired at a piece of tissue paper and bouncing back (quoted by Gentner and Wolff, 2000, p. 316; Randall, 2005, p. 127). His focus on the trajectory of the alpha particles passing through the foil target is confirmed by his frequent use, in his 1911 publication analyzing the experiment, of the analogy of a "pencil" of particles passing through a solid material. These analogies reflect the then-accepted idea that "such swiftly moving particles pass through the atoms in their path," although neither explains why "the deflexions (*sic*) observed are due to the strong electric field traversed within the atomic system" (Rutherford, 1911, p. 669); artillery shells, after all, do not bounce off tissue paper. Rutherford was aware of a hypothetical analogy between electrons arranged around an atomic core and the rings arranged around Saturn that had been advanced by Hantaro Nagaoka in 1904, but this notion of atoms as disk-shaped was unhelpful for explaining Rutherford's backscattering data. A different analogy was needed to provide a theoretically-productive picture for the structure of the atom.

The analogy that Rutherford chose to explain his results was simple and natural. If atoms are bombarded not by alpha particles but by electrons, the cumulative electrostatic force of the many electrons bound within the atom deflects the trajectories of the incoming electrons, sometimes through large angles. Rutherford noted that electrons are deflected at large angles far more frequently than alpha particles, and employed the analogy alpha-particle:central-charge::electron:atom, where "central-charge" was an hypothesized, uncharacterized point-like charged object contained within the gold atom, to calculate the observed alpha-particle deflections (Rutherford, 1911). Rutherford treated the mass of the central charge as negligible, estimated its electrostatic potential to be about 100 times that of an electron (the actual electrostatic charge of a gold nucleus is +79), and concluded that "the α particle must approach much closer to the center of the atom than the β particle (i.e. electron) of average speed to suffer the same large deflexion" (Rutherford, 1911, p. 687). Rutherford did not, in his 1911 paper, specify whether the "central charge" within the gold atom was positive or negative, use the term "nucleus" to refer to the central charge, or consider the electrons in the atom to be moving in orbits. Neither the "Rutherford atom analogy" electrons:nucleus::planets:sun nor the alpha-particle:nucleus::comet:sun analogy that has been suggested (e.g. Gentner and Wolff, 2000; Dietrich, 2010) as a motivator of Rutherford's reasoning appear in or are even suggested by his 1911 published analysis.

Three reasons can be suggested to explain why Rutherford did not employ the analogy that now bears his name in his published analysis of the alpha-particle scattering data. First, the received view from which Rutherford began his analysis was the Thompson model, in which atoms are solid objects. The mystery was not how particles fired at a Thompson atom could bounce off – electrons did bounce off – but how an "artillery shell" such as a high-velocity alpha particle could bounce off. This was not a qualitative mystery; it was a quantitative mystery that demanded a quantitative solution. The alpha-particle:central-charge::electron:atom analogy that Rutherford employed provides both an intuitive sense of the alpha particle's deep penetration into the target atom (at least for Rutherford's audience of physicists) and a direct basis for quantitative calculations. Second, while the analogy electrons:nucleus::planets:sun might suggest that the alpha particle is bouncing off something much heavier than an electron, it assumes the concept of an atomic "nucleus," a concept that did not exist, other than in Nagaoka's "Saturnian" model, until Rutherford derived it. Mass, moreover, was not the issue. As Rutherford pointed out at the very beginning of his 1911 paper, the masses of the objects involved are irrelevant; only their electric charges matter. Neither Rutherford nor his audience needed an analogy to suggest that electric charge was somehow "like" mass; the fact that the electrostatic and gravitational forces both obeyed inverse-square laws was well-established common knowledge. Third and most compellingly, the analogy electrons:nucleus::planets:sun suggests strongly that the electrons *orbit* the nucleus; indeed REVOLVE-AROUND is one of the relational concepts on which the analogy depends in its standard reconstruction. Electrons are charged particles, and according to classical electrodynamics charged particles moving in the vicinity of an electrostatic field continuously radiate energy. No such continuous radiation had ever been observed from atoms, and the Thompson model was carefully designed to accommodate this fact. It is therefore not surprising that Rutherford, in his paper showing that the Thompson model could not be correct, carefully avoided any implication that the electrons located on the periphery of his new-model atoms were moving.

The puzzling question of what the electrons were doing within an atom with a central charge was addressed by Bohr (1913), who showed that electrons could orbit the nucleus only if their orbits were quantized, through some unknown mechanism, to prevent the continuous radiation of their orbital energy and subsequent collapse of their orbits. Bohr's analysis, like Rutherford's, was motivated by an experimental result. Electrons in atoms were known to emit radiation only in discrete amounts, and only if excited by being irradiated themselves. Rutherford's characterization of the central point-like charge (i.e. the nucleus) provided an electrostatic field in which electrons could move and hence radiate, but the motion of the electrons and why they would radiate only at discrete energies remained mysterious. Bohr was faced with conflicting facts: electrons could emit radiation only by moving, but would emit radiation constantly if they moved in elliptical orbits like planets around the sun, on planar orbits like the disks of Saturn, or on any other classical trajectories. His response was the novel postulate of orbital quantization, and the radical notion that electrons in quantized "orbits" *do not move*, or at any rate do not move in any way that would count as "motion" in classical electrodynamics. Indeed, Bohr stated explicitly that "there obviously can be no question of a mechanical foundation" for his quantized model of electron orbitals (Bohr, 1913, p. 15). Hence while Bohr attributed to Rutherford the claim that "atoms consist of a positively charged nucleus surrounded by a system of electrons kept together by attractive forces from the nucleus" (Bohr, 1913, p. 1), he attributed no theory of electron motion to Rutherford, and made no mention of electrons:nucleus::planets:sun, the primary implication of which Bohr's quantum theory of atomic structure was explicitly designed to negate.

Besides the scholarly conundrum of whether Rutherford ever proposed or believed the "Rutherford atom analogy," this thumbnail history raises two intriguing questions. The first, clearly, is how the analogical reasoning that *did* occur was implemented in the brains of the scientists who performed it. In particular, how did Rutherford implement the key inference that alpha-particle backscattering was analogous to electron backscattering? The second question is why, given that its primary implication of classical electron orbits is wrong, the "Rutherford atom analogy" survives today. One might expect that if REVOLVE-AROUND(electrons,nucleus) is known to be false, a structure mapping between it and the true statement REVOLVE-AROUND(planets,sun) would be rejected by the presumably domain-general prefrontal (Wendelken *et al*., 2008; Knowlton and Holyoak, 2009) system that evaluates analogies for systematicity and hence explanatory power. This does not happen; physicists who know better still *teach* electrons:nucleus::planets:sun, and find that it benefits students (e.g. Podolefsky and Finkelstein, 2006). An adequate account of the implementation of electrons:nucleus::planets:sun must be capable of explaining why it is so compelling.

Force-motion analogies in the EFM model

Tool Improvisation and the EFM model of structure mapping

A very broad range of creatures routinely implement structure-mapping analogies of a particular kind: those enabling the improvisation of tools (Fields, 2011a). Improvising a tool requires an inference from a previous situation in which a goal was achieved using a particular tool – in the most basic case, an animal's own limbs – to a current situation with a similar goal that cannot be achieved with any available tool. For example, a chimpanzee may be faced with an edible nut that cannot be extracted from its shell by hand, or a human backpacker may be faced with a tent stake that cannot be driven into the ground with only a lightweight hammer. Success is achieved in such situations if an object – an improvised tool – can be found that enables a different force to be applied using a bodily motion similar to that used in the remembered previous situation; in the examples given, a suitable stone held in the hand enables the hand's motion to deliver a larger force. Tool-improvisation inferences are not, however, based simply on similarity of motion; the tool-dependent *relation* between motion and force is what is critical. Tool improvisation inferences are, therefore, relation-driven structure mappings organized by a five-part relation involving a tool, a target, a motion, a force, and a goal. The criterion for success of such inferences is not merely formal consistency or plausibility; it is the observable outcome of a real-world test. For tool-improvisation analogies, systematicity requires a *quantitatively* correct relation between motion and force.

Fields (2011a) proposed that tool improvisation analogies are implemented architecturally by the pre-motor action planning system in all species that improvise tools, and developed an "event-file manipulation" (EFM) model of tool improvisation based on two hypotheses well-supported by available data: 1) that both observed situations and re-instated memories involving tool use are represented in relational form by event files; and 2) that the pre-motor action planning system is capable of manipulating such event files to develop implementable action plans that involve the use of improvised tools. Event files, the data structures proposed by the EFM model, are transient multi-modal bindings of objects and motions with goals and action plans; they represent situations in which goal-appropriate action is being, has been or can be taken (reviewed by Hommel, 2004). Event files are continuously constructed and updated during attentive perception, and are reconstructed during episodic memory recall (Hommel, 2007; Keizer *et al*., 2008; Spapé and Hommel, 2010); in humans, they are implemented by distributed temporal-parietal-frontal activations that couple object feature and motion representations in ventral and medial-dorsal temporal cortex (Martin, 2007; Mahon and Caramazza, 2009) to the pre-motor "praxis network" that includes areas of parietal, cingulate, and both lateral and medial frontal cortex (Johnson-Frey *et al*., 2005; Culham and Valyear, 2006; Martin, 2007). The EFM model proposes that event files encoding situations involving tool use have a particular form: the **action**(tool,target,motion,force,goal) form required to represent the relational structure of tool use. Encoding as an event file requires that the "tool" and "target" roles in this relation be filled by modal (typically visual) images of objects, that the "motion" role be filled by a modal image of a three-dimensional trajectory, that the "force" role be filled by a motor image of the muscular forces required to maintain the "tool" on the trajectory specified by the "motion," and that the "goal" role be filled by a motivational representation implementable by cingulate and frontal cortex.

The second hypothesis of the EFM model is that the pre-motor action-planning system is capable of 1) retrieving memories of tool-use situations and re-activating them as event files; 2) comparing current to re-activated event files in working memory; 3) computing the forces required to maintain a specified trajectory; and 4) updating the "tool" role in the current event file with a specification for the required tool. The fact that tool-improvisation analogies yield tool specifications that explicitly reflect force computations is confirmed by two related observations. First, not only humans but also other tool-using animals engage in searches for objects that satisfy such force-related criteria as weight, hardness, length and stiffness (e.g. Carvalho *et al.*, 2008; Brill *et al.*, 2009; Visalberghi *et al.*, 2009). Second, both humans and other animals modify found objects to enhance such characteristics (Weir and Kacelnik, 2006; Pruetz and Bertolani, 2007). This second hypothesis of the EFM model is consistent with, and indeed a special case of, the broader hypothesis that the pre-motor system is a general-purpose anticipation and prediction system that computes sufficient information to enable anticipation-driven actions (Butz *et al.*, 2003; Schubotz, 2007; Bubic *et al*., 2010).

The EFM model only characterizes the *implementation* of structure mapping; it makes no claims regarding an animal's *understanding* of objects, motions, forces, or the analogies that may hold between them. Substantial data suggest that non-human animals lack any human-like understanding of tools or tool-improvisation analogies (Penn and Povinelli, 2007; Suddendorf and Corballis, 2007; Penn *et al.*, 2008). Human beings do understand analogies, and often do so using language. How this language-based understanding is implemented is not well characterized.

Reconstruction of alpha-particle:central-charge::electron:atom within the EFM model

The primary hypothesis of the present paper is that the EFM model is sufficient to implement all force-motion analogies, even those in abstract domains such as atomic physics. This hypothesis has two components. First, it is proposed that event files are a fully adequate data structure for the representation of physical situations involving objects, motions and forces, even if the objects and motions are not directly observable and must instead be imagined. Second, it is proposed that the pre-motor action planning system implements structure mappings over event files encoding general object, motion and force relations in the same automated, architecturally-supported way that it implements structure mappings for tool improvisation.

To examine this hypothesis, consider the implementation of Rutherford's published analogy alpha-particle:central-charge::electron:atom. The observational bases for this analogy are data recording the angles with respect to the incident beam at which backscattered electrons or alpha particles were detected; each of these recorded angles corresponds to a particle trajectory that begins at the electron or alpha-particle source, is bent by the measured angle at the target, and ends at the detector. Physicists routinely employ both drawings and internal visual images to represent objects such as alpha particles, electrons and atoms, and the trajectories that they follow in the course of interactions (Randall, 2005 provides many examples). Substantial evidence indicates that internal visual images are implemented by the same networks that implement visual perceptual processing (reviewed by Kosslyn *et al.*, 2006; Moulton and Kosslyn, 2009); such images therefore generate event files (Hommel, 2004; Spapé and Hommel, 2010). Two sets of trajectories, one for electrons and the other for alpha particles, represented as internal visual images can, therefore, be taken to be Rutherford's starting point for explaining how “artillery shells” bounce back from gold foils. Rutherford's goal is to find a force capable of deflecting alpha particles backwards along trajectories similar to those of electrons deflected backwards by atoms.

The EFM model as presented (Fields, 2011a) assumes that the target of an action is stationary. The model can be extended to accommodate moving targets provided that the action planning system is capable of representing relations of the form **action**(tool,target(motion,force),motion,force,goal) in which a force-delivering motion is attributed to the target. As EFM representations of forces are motor images, embedding motion and force attributes of the target requires representation of the observed motion of the target in terms of the motor forces required to replicate that motion. Such “mirror” functionality is well developed in primates (reviewed by Puce and Perrett, 2003; Rizzolatti and Craighero, 2004; Culham and Valyear, 2006; Cattaneo and Rizzolatti, 2009). The human mirror system is known to be responsive to non-biological motion patterns (Schubotz and von Cramon, 2004; Engel *et al.*, 2007) and to exhibit changes in specificity in response to perceptual experience (Catmur *et al*., 2007; 2008; 2009; reviewed by Heyes, 2010). The representation of actions involving moving targets required by the EFM model is, therefore, consistent with available data regarding human mirror system capabilities.

The final requirement for the implementation of Rutherford's analogy alpha-particle:central-charge::electron:atom within the EFM model is that both Rutherford and his intended audience would be capable of constructing motor representations that differentiated the large force propelling the incoming alpha particle from the much smaller force propelling the incoming electron. People with no formal instruction in physics are capable of solving mechanical problems involving differential forces given before-and-after images or simple descriptions; their typical inability to explain how they have solved such problems is *prima facie* evidence that they do so non-verbally (Pinker, 1997; Hegarty, 2004; Wolff, 2007; 2008; White, 2009). One objective of instruction in physics is to instill such "intuitions" in progressively more esoteric domains, a process that is considerably aided by the use of non-verbal teaching modalities (Podolefsky and Finkelstein, 2006; Lasry and Aulls, 2007).

Given the considerations above, Rutherford's alpha-particle:central-charge::electron:atom analogy can be implemented within the EFM model as shown in Fig. 1. Starting from his knowledge that atoms exert a force that deflects incoming electrons, and that alpha particles exhibit similar deflections, Rutherford could infer by a structure mapping defined over event files both that something deflected the alpha particles by exerting a force on them and that the something, whatever it was, was capable of exerting a strong force. By assuming that the forces involved were electrostatic, Rutherford reached his critical insight into atomic structure: that a *large electric charge* was concentrated at the center of the atom (Rutherford, 1911).

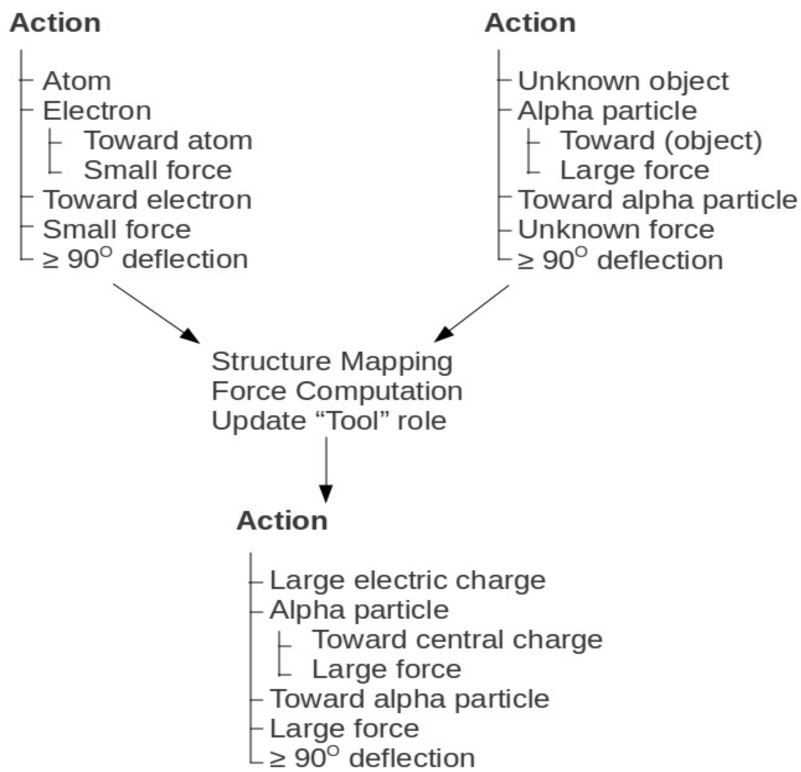

Fig. 1. Illustration of steps in Rutherford's 1911 analogical inference that atoms contain a central concentration of electric charge. Event files are represented as relations of the form **action**(tool,target(motion,force),motion,force,goal), in which the "tool" and "target" roles in this relation are filled by modal images of objects, the "motion" roles are filled by modal images of three-dimensional trajectories, the "force" roles are filled by motor images of the muscular forces required to maintain the "tool" or "target" on the trajectory specified by its "motion," and the "goal" role is filled by a motivational representation implementable by cingulate and frontal cortex. The use of English labels to represent these relata is heuristic only and is not meant to imply that the pre-motor action system employs symbols representing relational concepts specified in either public language or an internal "language of thought."

Dietrich (2000; 2010) has emphasized that productive analogical reasoning involves not only structure mapping, but also the generation of novel interpretations or “construals” of current data (see also Blanchette and Dunbar, 2002). The inferential process illustrated in Fig. 1 clearly involves such an interpretative step; the initially unknown object that deflects the incoming alpha particle is construed as a large electric charge since the force calculation indicates that it must exert a large force. The EFM model predicts that, within the domain of force-motion analogies, such construals will in general be specifications of the kind of object that can deliver a required force.

Rutherford's other force-motion analogies

If the EFM model is correct, other force-motion analogies should also be implementable using the specified data structures and the inferential form illustrated in Fig 1. The alpha-particle:foil-target::artillery-shell:tissue-paper analogy Rutherford reported to colleagues and the alpha-particle:foil-target::pencil:foil-target analogy he published both involve stationary targets and reflect Rutherford's observation that the vast majority of alpha particles punch through the gold-foil target without being acted upon by any resistive force. They can both be viewed as straightforward instances of tool-improvisation analogies with the alpha particle in the role of the tool. It is the analogies that Rutherford did not publish but that are nonetheless commonly attributed to him, alpha-particle:nucleus::comet:sun and electrons:nucleus::planets:sun, that pose a greater challenge to the model.

Rutherford's 1911 paper introduced the notion of a central charge that would, by Bohr's paper in 1913, be called the “nucleus” of an atom. Prior to Rutherford's realization that a central charge existed, the analogy alpha-particle:nucleus::comet:sun could only be framed as alpha-particle:*atom*::comet:sun or possibly alpha-particle:atom::comet:solar-system. Neither of these formulations passes the test of systematicity: almost all alpha particles pass straight through atoms without deflections, while most known comets orbit the Sun. Following Rutherford's discovery, however, inferring alpha-particle:nucleus::comet:sun only required knowing that the gravitational and electrostatic forces have the same mathematical form, knowledge that any physicist in 1911 would have had. Hence the EFM model predicts that the alpha-particle:nucleus::comet:sun analogy *derived from* the alpha-particle:central-charge::electron:atom analogy by a structure mapping that related the central charge in the atom to the Sun and the electrostatic to the gravitational force. Such a derivative structure mapping could have been implemented entirely conceptually, as the standard model of analogy (Gentner, 2003; Holyoak, 2005) would suggest.

Unlike alpha-particle:nucleus::comet:sun, electrons:nucleus::planets:sun does not appear to concern alpha particle trajectories at all. As pointed out earlier, if electrons:nucleus::planets:sun is interpreted as a claim about the motion of electrons, it fails the test of systematicity. If electrons:nucleus::planets:sun is interpreted only as a claim about the *distance* between the electrons and the nucleus, it leaves the knowledge that planets move in orbits and that the Sun attracts them unmapped and hence has poor systematicity. In either case, formulating electrons:nucleus::planets:sun requires having some pre-existing concept of a nucleus; it makes no sense at all within the Thompson model of the atom. Hence electrons:nucleus::planets:sun is also predicted to be derivative from, not a motivator of, alpha-particle:central-charge::electron:atom.

What is most intriguing about electrons:nucleus::planets:sun, however, is not its provenance but its iconic status. The interpretation of electrons:nucleus::planets:sun within the analogy community as a structure mapping based on REVOLVE-AROUND is not idiosyncratic, but consistent with its pedagogical use in numerous physics and chemistry textbooks. The fact that this analogy, the one among all of Rutherford's with a primary implication that is straightforwardly false, survives while the others have lapsed into obscurity suggests that the notion of orbiting electrons is compelling in a way that the notion of deflected alpha particles is not. Until very recently, human beings have directly experienced only one exemplar of orbital motion: the centrifugal force felt when swinging a weight attached to a tether. The survival of electrons:nucleus::planets:sun in the face of all evidence to the contrary suggests that this motor memory of force and motion in fact implements the "Rutherford atom analogy."

Conclusion

Structure mapping analogies are inferences defined over relations. The plethora of relational terms in natural languages, the common expression of analogies in language, and the facility with which structure mapping can be implemented using relational terms as lexical symbols have contributed to the standard view that structure mappings are implemented by humans, consciously or unconsciously (Day and Gentner, 2007; Day and Goldstone, 2011), using a "mentalese" with the structure and semantics of a natural language. Analyses and reconstructions of historically-significant analogies in the sciences have accordingly assumed an implementation of structure mapping based on relational concepts expressed in language and manipulated as symbols (Gentner *et al.*, 1997; Gentner and Wolff, 2000).

The observation that non-human animals execute non-trivial structure mappings when improvising tools raises the possibility that structure mapping is an ancient and ubiquitous inferential mechanism implemented by processes that do not manipulate lexical symbols representing relations. The EFM model proposes a specific implementation of structure mapping by the pre-motor action-planning system (Fields, 2011a). As shown here, the EFM model provides both the representational capacity and inferential capability required to implement structure mappings in the force-motion domain. As an ability to perform rapid inferences concerning forces and motions is a general requirement for mobile animals, the implementation of such abilities by the pre-motor system is evolutionarily plausible.

An adequate experimental test of the EFM model will require either high-resolution differentiation between functional networks involved in trajectory recognition and motor planning as compared to language processing or an understanding of the primate implementations of trajectory recognition and motor planning sufficient to construct realistic computational models. Given the current high level of interest in these functions (Bubic *et al.*, 2010; Fields, 2011b), such an understanding may soon become available.

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